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Evaluation of Parameter Estimation Methods for Unstable Aircraft

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While continuing to be of wide interest, the estimation of aerodynamic derivatives from flight data of an unstable aircraft poses several difficulties. The objective of this article is to provide an overview of some of the more recently introduced estimation methods together with some of the conventional ones. Five options addressed here are 1) regression startup, 2) equation decoupling, 3) filter error, 4) output error with artificial stabilization, and 5) multiple shooting method. An evaluation is made based on the parameter estimates both from simulated and flight data. The various methods yield comparable estimates from simulated responses pertaining to the short period motion of an unstable aircraft. Their application to X-31A aircraft flight data in unstable flight regime brings out some practical aspects. It is found that the application of the conventionally used output error method with artificial stabilization and of the multiple shooting method requires considerable engineering effort, and can still pose difficulties when analyzing longer duration maneuvers. The estimates from the filter error and equation decoupling methods compare with each other well. It is demonstrated that they provide an attractive and less laborious alternative to analyze longer duration maneuvers, e.g., consecutive elevator doublets or sweep inputs up to 30 s.

Introduction

THE demands of improved performance characteristics, e.g., high maneuverability of modern flight vehicles have led to aerodynamically unstable aircraft configurations. Advancements in modern control theory, microelectronics, and computer techniques have helped to make such basically unstable aircraft flyable. Although unstable aircraft can be flown only with the aid of a flight controller, i.e., in closed loop, the prediction and flight determination of aerodynamic derivatives of the basic unstable aircraft, i.e., of the open loop plant, is of primary interest for many applications in the field of flight mechanics. The importance of validating the wind-tunnel and theoretical estimates and, if necessary, updating these predictions with those estimated from flight data is well recognized.[1,2] This is particularly true in the case of new aircraft designs such as the X-31A, where the results of flight data analysis are used for flight envelope expansions.[3,4]

The system identification methods, both in time and frequency domain, have been successfully applied during the past few decades to determine aerodynamic parameters from flight data, at least of stable aircraft configurations. The frequency domain identification methods can be readily extended to unstable systems.[1] These methods are, however, limited only to linear equations of motions. This constitutes a major disadvantage in practical applications of these methods, particularly for modern flight vehicles often flying in extreme flight regimes exhibiting nonlinear aerodynamic characteristics. Therefore, this article concentrates on the time domain identification methods.

In the case of time domain methods, numerical integration of unstable and highly sensitive systems is the major problem which leads to a diverging solution, more often than not exceeding the numerical range of computers.[5,6] Hence, special techniques and modifications are necessary to prevent the growth of errors introduced by poor initial values, round off, or discretization, and propagated by inherent instabilities of the system equations. Such approaches are based either on limiting the integration interval or making more efficient use of the observed data.

As evident from the literature, the output error method, which accounts for measurement noise in output variables, is the most commonly applied method to estimate the aerodynamic derivatives from flight data.[1,2] It is applicable to both linear and nonlinear systems.[7,8] It is, in principle, also applicable to unstable systems. Such attempts, however, encounter the aforementioned numerical difficulties, making direct application of the output error method to unstable systems tedious, if not impossible, requiring extensive engineering effort. To provide a practical solution, by analogy to the filter error methods possessing stabilizing properties, the output error method was extended to provide artificial stabilization.[6,9] This ad hoc approach has been applied by a few investigators with some success to flight data analysis. In those applications, however, it was necessary to restrict the maneuver, i.e., the time segment, being analyzed to a short record length, typically 5 s. Its application to a longer duration maneuver such as sweep input maneuver or acceleration and deceleration maneuver, without time segmenting, was not reported.

The filter error and least squares methods are more readily applicable to unstable systems. The use of measured states in the least squares approach stabilizes the numerical solution. The filter error methods, incorporating a state estimator consisting of a prediction and update step, possess inherent stabilizing properties. However, these two methods have hitherto not seen wide applications. It is because the least squares approach is known to yield biased estimates in the presence of measurement errors in regressors, whereas the filter error methods are complex and were until recently restricted mainly to linear systems. This article explores in some depth their applications to unstable systems with a view that a practical

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solution to extension of filter error methods to nonlinear systems is possible,[10] and that the cpu time requirement plays only a minor role in the overall analysis. In the case of least squares methods,[11] the high quality sensors and instrumentation minimizes the errors on the one hand, and on the other hand determination of the systematic error applying data compatibility check is only a minor additional burden.

The equation decoupling method makes use of the measured state variables to reformulate the system equations, and integrating the differential equations independent of each other. This approach was recently validated on simulated data and introduced as an alternative to the output error method with artificial stabilization.[12] It is shown that this approach is based on the basic principles of regression analysis and applied to real flight data in this article.

The multiple shooting approach is an efficient technique for the solution of two-point boundary value problems.[13] Although the general approach has been recognized for several decades, its application to parameter estimation problems has rarely been attempted.[14,15] Its applicability to simulated, as well as flight data, is explored here.

After providing a brief overview of the parameter estimation methods for unstable systems, they are validated on the simulated short period motion of an unstable aircraft. In the second part of the data analysis, they are applied to the flight data of the X-31A aircraft in the unstable flight regime. The main focus is on estimating aerodynamic parameters from longer duration maneuvers, such as sweep input maneuver, without time segmenting. A comparison of the estimation results is made, bringing out practical aspects and advantages as well as limitations of the methods investigated.

Model Formulation

The dynamic system, whose parameters are to be estimated, is assumed to be described in the state space as

$$\dot{\boldsymbol{x}}(t) = f[\boldsymbol{x}(t), \boldsymbol{u}(t) - \Delta\boldsymbol{u}, \boldsymbol{\beta}], \qquad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{1}$$

$$y(t) = g[\boldsymbol{x}(t), \boldsymbol{u}(t) - \Delta\boldsymbol{u}, \boldsymbol{\beta}] + \Delta z \tag{2}$$

$$z(t_k) = y(t_k) + v(t_k) \qquad k = 1, \ldots, N \tag{3}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ the control input vector, y the observation vector, and $\boldsymbol{\beta}$ the vector of unknown system parameters. The system functions f and g are general nonlinear real valued vector functions, and are assumed to have sufficient differentiability. The measurement vector z is sampled at N discrete time points with a uniform sampling time of Δt. The measurement noise vector v is assumed to be characterized by a sequence of independent Gaussian random variables with zero mean and covariance matrix R.

In addition to the system parameters $\boldsymbol{\beta}$, the initial conditions $\boldsymbol{x}_0$ and the systematic errors $\Delta\boldsymbol{u}$ and Δz are usually unknown. It is thus, in general, required to estimate

$$\Theta^T = (\boldsymbol{\beta}^T, \boldsymbol{x}_0^T, \Delta\boldsymbol{u}^T, \Delta z^T) \tag{4}$$

from the discrete measurements of the system response $z(.)$ to given control inputs $\boldsymbol{u}(.)$ subject to the model postulated in Eqs. (1–3). Assuming that the measurement error covariance matrix R is known, the maximum likelihood estimate of Θ is obtained by minimizing the cost function

$$J(\Theta) = \sum_{k=1}^{N} [z(t_k) - y(t_k)]^T R^{-1} [z(t_k) - y(t_k)] \tag{5}$$

with respect to Θ. Starting from the suitably specified initial values of Θ, the new updated estimates are obtained by applying the Gauss-Newton method[7]

$$\Theta_{i+1} = \Theta_i + \lambda \Delta\Theta \tag{6}$$

$$\Delta\Theta = \left\{ \sum_k \left[\frac{\partial y(k)}{\partial \Theta}\right]^T R^{-1} \frac{\partial y(k)}{\partial \Theta} \right\}^{-1} \times \left\{ \sum_k \left[\frac{\partial y(k)}{\partial \Theta}\right]^T R^{-1} [z(k) - y(k)] \right\} \tag{7}$$

where λ is the damping factor. The sensitivity matrix $[\partial y/\partial\Theta]$ is obtained by numerical approximations.[8,10]

Although it was assumed here that R is known, it is possible to obtain its maximum likelihood estimate at only a minor computational burden.[7] In the examples analyzed and reported in this article, the error covariance matrix was estimated.

Parameter Estimation Methods

In the following a brief description of the five estimation methods investigated in this article is provided. They employ different techniques with different justifying arguments and with varying degrees of complexity to overcome the aforementioned numerical difficulties. These time domain methods are applicable to both linear and nonlinear systems. In some cases the linearized form of the system Eqs. (1–2) is used, but only for the sake of clarity.

Regression Startup Method

The regression startup method belongs to the broad class of least squares method. To illustrate the method, consider the linearized system representation:

$$\dot{\boldsymbol{x}}(t) = A(\boldsymbol{\beta})\boldsymbol{x}(t) + B(\boldsymbol{\beta})\boldsymbol{u}(t) \tag{8}$$

$$y(t) = C(\boldsymbol{\beta})\boldsymbol{x}(t) + D(\boldsymbol{\beta})\boldsymbol{u}(t) \tag{9}$$

Now, assuming that the independent measurements of all the state variables $\boldsymbol{x}_m$ are available, the system equations are reformulated as[11]

$$\dot{\boldsymbol{x}}(t) = A_F\boldsymbol{x}(t) + [B(\boldsymbol{\beta});\ A_E(\boldsymbol{\beta})] \begin{bmatrix} \boldsymbol{u}(t) \\ \boldsymbol{x}_m(t) \end{bmatrix} \tag{10}$$

$$y(t) = C_F\boldsymbol{x}(t) + [D(\boldsymbol{\beta});\ C_E(\boldsymbol{\beta})] \begin{bmatrix} \boldsymbol{u}(t) \\ \boldsymbol{x}_m(t) \end{bmatrix} \tag{11}$$

The state matrices A and C are resolved into two matrices each, namely (A_F; A_E) and (C_F; C_E). The matrices with the subscript F contain the possible fixed coefficients, if any, and those with the index E contain the parameters to be estimated.

This reformulation makes the state variables $\boldsymbol{x}$, and thereby the observation variables y, and hence the error $z(t_k) - y(t_k)$ linear-in-parameters being estimated. For such cases, the cost function Eq. (5) being quadratic, the regression method can be easily applied. This formulation, often called as regression startup, has the advantage that the initial values of the unknown parameters are not required. The optimization procedure can be started with zero values of all the parameters, at least for stable systems.

Equation Decoupling Method

The basic idea behind this method is, with the aid of independent state measurements, to reformulate the state model in such a way that each differential equation can be integrated independently.[12] Continuing with the linear system representation of Eq. (8), partition the system matrix A into two submatrices denoted by A_D and A_{OD}, where A_D is diagonal containing the diagonal elements of A and the matrix A_{OD} the off-diagonal elements. Now, augmenting $\boldsymbol{u}$ with the measured states $\boldsymbol{x}_m$, Eq. (8) can be rewritten as

$$\dot{\boldsymbol{x}}(t) = A_D(\boldsymbol{\beta})\boldsymbol{x}(t) + [B(\boldsymbol{\beta});\ A_{OD}(\boldsymbol{\beta})] \begin{bmatrix} \boldsymbol{u}(t) \\ \boldsymbol{x}_m(t) \end{bmatrix} \tag{12}$$

Since the reformulated system is decoupled, as implied by A_D, it changes the original unstable system into a decoupled stable system. Although, strictly speaking, such a reformulation does not guarantee a stable decoupled system, the current experience and those reported in Ref. 12 indicates that it does lead to stable systems in practical cases. It is possible to consider various degrees of decoupling, depending upon the resolution of matrix A.

Application of this method clearly requires, as already pointed out, independent measurements of the state variables. Furthermore, they must be noise free, otherwise the reformulation would introduce a process noise component in the decoupled state equations. For this reason, application of the method optimizing the output error cost function may require some consideration in a few cases, e.g., when the postulated model contains the angle of attack or of sideslip as a state variable. These vane measurements are usually less accurate and noisy.

The method, although illustrated on a linear system, is equally valid and applicable to nonlinear systems. The equation decoupling method, as it becomes evident, is a variation of the regression method commonly incorporated as a startup procedure for the output error method.[11]

Filter Error Method

The filter error approach to parameter estimation is probably the most general one.[7,10] It is computationally complex, and until recently, restricted mainly to linear systems. Although it remains computationally complex, the breathtaking advancements in computer technology have partially neutralized the weightage placed on the disadvantage of requiring more cpu time compared to other methods. In fact, in any practical exercise on parameter estimation the actual cpu time is only a minor part of the total time, the major part being consumed by mundane tasks such as checking the time histories, collecting and analyzing the results, and generating the plots, etc. The filter error methods have also been recently extended to nonlinear systems.[10] A nonlinear constant gain filter incorporated in the maximum likelihood estimation is found to provide a practical solution. A numerical approximation of sensitivity coefficients enables development of the estimation software without requiring programming modifications each time the nonlinear model structure is changed. The filter error approach basically caters to the need of analyzing flight data in turbulent atmosphere. In this case the state model postulated in Eq. (1) is extended to include the unmeasurable, stochastic input

$$\dot{x}(t) = f[x(t), u(t) - \Delta u, \beta] + Fw(t), \qquad x(t_0) = x_0 \tag{13}$$

where w is the process noise, which is characterized by a zero-mean white Gaussian noise with an identity power spectral density. F denotes the distribution matrix. The cost function defined in Eq. (5) gets modified to

$$J(\Theta) = \sum_{k=1}^{N} [z(t_k) - \tilde{y}(t_k)]^T \tilde{R}^{-1} [z(t_k) - \tilde{y}(t_k)] \tag{14}$$

where $\tilde{y}$ is the predicted observation vector and $\tilde{R}$ the covariance of the innovations (residuals). A nonlinear constant gain filter, consisting of a prediction and correction step, provides $\tilde{y}$ over the time length being analyzed.

Prediction

$$\hat{x}(t_0) = x_0 \tag{15}$$

$$\tilde{x}(t_k) = \hat{x}(t_{k-1}) + \int_{t_{k-1}}^{t_k} f[x(t), u(t) - \Delta u, \beta]\, dt \tag{16}$$

$$\tilde{y}(t_k) = g[\tilde{x}(t_k), u(t_k), \beta] \tag{17}$$

Correction

$$\hat{x}(t_k) = \tilde{x}(t_k) + K[z(t_k) - \tilde{y}(t_k)] \tag{18}$$

where $\tilde{x}$ is the predicted state vector, $\hat{x}$ the updated state vector, and K the filter-gain matrix.

The steady-state K is a function of the steady-state R, the steady-state P, and the observation matrix C ($= \partial g/\partial x$) of the linearized system. The state prediction error covariance matrix P is obtained by solving the Riccati equation. A detailed description of the filter error methods is found in Refs. 7, 10, and 16.

The state estimators incorporate, as evident from Eq. (18), the measurements of the observation variables. A feedback proportional to the fit error updates the predicted state variables. This feedback stabilizes numerically the filter algorithm and also helps to improve the convergence properties of the estimation algorithm. This stabilizing property of the filter error algorithm can be exploited to analyze unstable systems. As pointed out in Ref. 6, the filter feedback need not bear any relation to the control system feedbacks that actually help to fly the unstable aircraft.

Output Error Method with Artificial Stabilization

The output error method is undoubtedly the most commonly used parameter estimation method for stable systems.[1,2] It is applicable both to linear and nonlinear systems, and has several desirable statistical properties.[7] Its application to unstable systems encounters many practical difficulties, not due to the theoretical formulation but mainly due to numerical computations. It is well known that the solution of an unstable system is sensitive to the errors in the initial conditions and system parameters. In our case since the aerodynamic parameters are unknown, in addition to the unknown or inaccurately known initial conditions, it leads to a diverging integrated solution, more often than not exceeding the numerical range of a computer. This behavior is caused by characteristic attributes of the single shooting approach, i.e., of the initial-value problems, incorporated in the output error method.

To overcome these practical difficulties, by analogy to the filter error methods possessing stabilizing properties, the output error method is extended to provide artificial stabilization.[6,9] Although such an artificial stabilization is arbitrary and does not have the theoretical basis, the approach is seen as a practical solution to extend the commonly accepted output error method for parameter estimation to unstable aircraft.

Such an algorithm is parallel to the filter error algorithm of Eqs. (13–18), except that the updated state vector is now computed according to

$$\hat{x}(t_k) = x(t_k) + S[z(t_k) - y(t_k)] \tag{19}$$

where S is the arbitrarily selected stabilization matrix, assumed to be independent of the system parameters and initial conditions. This overcomes the need for computation of the filter gain matrix, which is the computationally complex and time-consuming part of the filter error method.

Two limiting values of S are the null and identity matrix. For $S = 0$ the algorithm corresponds to the output error method. In the other case of $S = 1$, it reduces to the equation error method. Introduction of the stabilization matrix S affects the parameter estimates. This influence, however, will be small provided the elements of S are selected to be small. Its influence will in any case be minimized when the output error is small, i.e., when modeling errors are small. Furthermore, the choice of the type of stabilization, i.e., which fit error to be fed back to which state variable, is also neither defined nor obvious. This may call for some engineering judgment.

In addition to stabilizing the system solution, stabilization of the solution to the sensitivity equations is also necessary. By partial differentiating Eq. (19), it can be shown that the same stabilization matrix is to be used to stabilize the perturbed state solutions used in numerically approximating the sensitivity coefficients.[9]

Multiple Shooting Method

The commonly applied technique to solve estimation problems is fitting the unknown parameters in an iterative procedure by solution of the initial-value problem (single shooting). In this approach, optimization of a scalar function does not make efficient use of the primary information available to the estimation procedure, which is defined by the measured trajectories. In contrast, the multiple shooting approach accounts more efficiently for the provided information.

The basic concept of the multiple shooting approach is to subdivide the integration interval $[t_0, t_f]$ by suitable choice of a grid

$$g^m: (t_0 = \tau_1 < \tau_2 < \cdots < \tau_m = t_f) \tag{20}$$

into $(m - 1)$ intervals (Fig. 1). The corresponding initial-value problems

$$\dot{x}(t) = f(t, x, \Theta), \qquad x(\tau_j) = \sigma_j, \qquad t \in (\tau_j, \tau_{j+1}) \tag{21}$$

with the additional variables $(\sigma_1, \ldots, \sigma_m)$ as estimates of the states $x(\tau_j)$ can be solved independently with classical integration methods, such as the Runge-Kutta algorithm. In order to ensure continuity of the final solution over the entire time interval, additional constraints

$$h_j(\sigma_j, \sigma_{j+1}, \Theta) = x(\tau_{j+1}|\sigma_j, \Theta) - \sigma_{j+1} = 0$$
$$j = 1, \ldots, (m - 1) \tag{22}$$

have to be satisfied. This technique transforms the original identification problem into a constrained least squares problem

$$\left|J(\tilde{\Theta})\right|_2^2 \rightarrow \min_{\tilde{\Theta}} \tag{23}$$

subject to the equality constraints of Eq. (22) and the extended unknown parameter vector $\tilde{\Theta} = [\sigma_1, \ldots, \sigma_m, \Theta]$.

A direct solution to this problem is provided by constrained optimization. A more efficient approach is, however, to recognize the special structure and to apply the Gauss-Newton method in combination with a condensation algorithm. A detailed description of the formulation is found in Refs. 13–15. The advantage of the multiple shooting method is that by defining additional grid points (σ_j), the computed solution remains close to the observed data. Exponential growth of the parasitic components in the solution of an unstable differential equation can be restricted. The choice of grid points is one critical aspect in the application of the multiple shooting method, influencing both estimates and the convergence properties. The grid points may be equidistant and can be either specified a priori or can also be determined automatically by specifying a tolerance limit for the error growth.

The foregoing approach of subdividing the integration interval into several segments is not the same as that of evaluating simultaneously multiple time records. In the multiple run evaluation applying output error method, the individual time segments are treated independently, whereas the multiple shooting approach enforces continuity of the solution at the grid points.

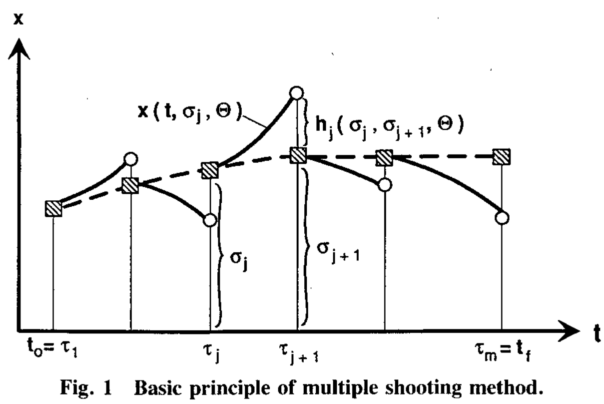

Fig. 1 Basic principle of multiple shooting method.

Simulated Unstable Aircraft Response Data

To evaluate the performance of the various estimation algorithms on unstable systems, typical aircraft responses are generated through simulation. A simplified model pertaining to the short period motion of the test aircraft de Havilland DHC-2 "BEAVER" is considered here.[17] To provide realistic control inputs, the pilot input actually applied in a particular flight is used. It corresponds to the optimized Mehra-Input signal.[17] A total of 12.5 s of data with a sampling time of 0.05 s has been generated.[9]

The nominal values of the aerodynamic derivatives used to generate the response data corresponds to those obtained by parameter estimation from flight data at a typical flight condition (Table 1). The static stability parameter M_w is, however, adjusted to result in an unstable system with time-to-double of 1 s. The state and observation equations are

State equations

$$\dot{w} = Z_w w + (u_0 + Z_q)q + Z_{\delta_e}\delta_e \tag{24a}$$

$$\dot{q} = M_w w + M_q q + M_{\delta_e}\delta_e \tag{24b}$$

Observation equations

$$A_z = Z_w w + Z_q q + Z_{\delta_e}\delta_e \tag{25a}$$

$$w = w \tag{25b}$$

$$q = q \tag{25c}$$

where w denotes the vertical velocity, u_0 the stationary forward speed, q the pitch rate, A_z the vertical acceleration, and δ_e the elevator deflection.

Numerical simulation of such an unstable aircraft is feasible only through incorporation of a suitable controller. A feedback proportional to the vertical velocity is used

$$\delta_e = \delta_p + kw \tag{26}$$

where δ_p denotes the pilot input. The feedback k is chosen to result in the original degree of stability for the closed loop system.[9] Although the data is generated through a closed loop simulation, it is attempted to identify the system in open loop, i.e., treating δ_e, the actual elevator position, as the control input. The controller is not considered in the estimation procedure.

The numerical difficulties encountered by the conventional output error method without any stabilization are illustrated in Fig. 2. This figure, taken from Ref. 9, was obtained by specifying all the parameters to a two-digit accuracy to avoid numerical overflows, possible only for this simulation example. The methods described in the foregoing section are now applied to estimate the dimensional derivatives appearing in Eqs. (24) and (25). The results are provided in Table 1.

The regression startup method, as already pointed out, does not require initial parameter values. Accordingly, the iterations were started with zero values for all the parameters. No numerical difficulties were encountered and the convergence was rapid, although the system being analyzed is unstable.

Table 1 Parameter estimates from simulated unstable aircraft responses

Parameter	Nominal value	Regression analysis	Equation decoupling	Output error with artificial stabilization	Multiple shooting	Filter error
Z_w	−1.4249	−1.4249	−1.4277	−1.4281	−1.4273	−1.4252
Z_q	−1.4768	−1.4768	−1.4768[a]	−1.4768[a]	−1.4849	−1.4706
$Z_{\delta e}$	−6.2632	−6.2632	−6.1681	−6.1582	−6.1734	−6.2605
M_w	0.2163	0.2163	0.2162	0.2174	0.2168	0.2184
M_q	−3.7067	−3.7121	−3.7085	−3.7212	−3.7129	−3.7484
$M_{\delta e}$	−12.784	−12.769	−12.773	−12.821	−12.802	−12.882

[a]Kept fixed in estimation due to resulting very high correlations.

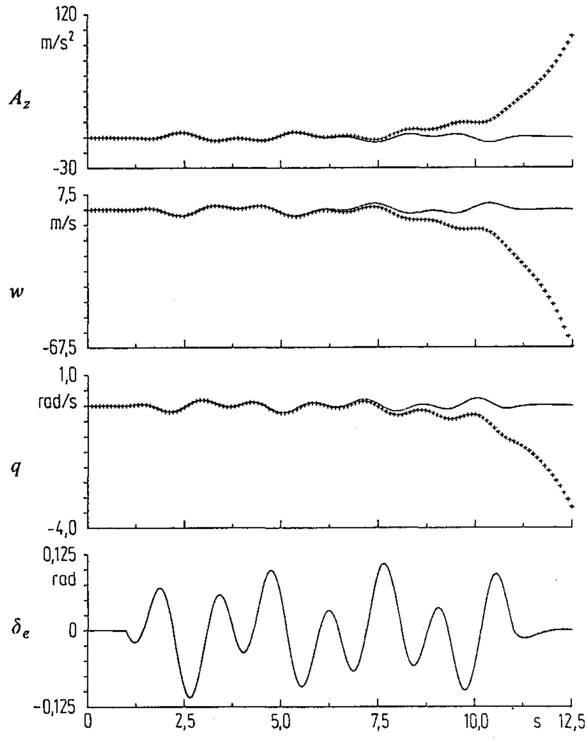

Fig. 2 Identification of unstable system in open loop (simulated data). Parameter values accurate to 2 digit decimal; —— simulation, + + + + estimated.

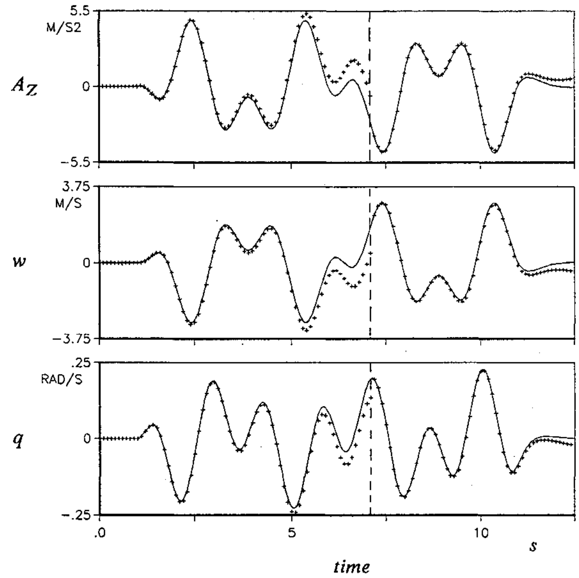

Fig. 3 Identification of unstable system in open loop (simulated data). Starting solution applying the multiple shooting method; —— simulation, + + + + estimated.

As seen from Table 1, the parameters Z_w, Z_q, $Z_{\delta e}$, and M_w are estimated exactly. Small deviations in the second digit after the decimal are noted in the estimates of M_q and $M_{\delta e}$, which are attributed to the slight correlation between the estimated parameters. This could possibly be related to several causes, such as type of maneuver being analyzed, input signal applied, finite digit data representation, or numerical approximations, etc. A good agreement between the estimated and nominal values is not really surprising, because the regression method is known to yield bias-free estimates in the absence of measurement noise, as was the case here with the simulated data.

The estimates obtained by applying the other methods tend to agree fairly well with the nominal values. In the case of the filter error method, the process noise distribution matrix was kept fixed in the estimation procedure at an extremely small value. This choice is equivalent to the nonzero covariance of the state prediction error,[6] and eliminates accounting for the unmodeled effect through process noise options. The convergence was achieved in 4–5 iterations. Although the iterations could not be started off with zero values for all the parameters, the convergence was not too sensitive to the initial values. The convergence region for this method was much larger compared to the output error method and to the equation decoupling technique.

In the equation decoupling and output error method, it was necessary to keep the derivative Z_q fixed at the nominal value. Attempts to estimate this derivative lead to extremely correlated estimates. For these two methods, starting values closer to the nominal values were required. Otherwise, numerical overflows were encountered. The equation decoupling method had a slightly larger convergence region than the output error method. In the case of output error with artificial stabilization, the fit error in pitch rate, i.e., $(q_m - q)$, was fed back to the state variable q with a factor of 0.02.

In the case of the multiple shooting method, a detectable dependence of the estimates and of the convergence properties on the chosen grid is observed. With reasonably good initial parameter values, one or two grid points were necessary to prevent the growth of the errors leading to numerical overflows, see Fig. 3. A convergence was achieved in 5–6 iterations. Better estimation results were, however, obtained by using every 15th data point as a grid point. This choice of the grid also led to a larger convergence region.

Figure 4 shows the comparison of the time responses applying a typical estimation method. The very good performance of the various methods being evaluated in this article on the simulated unstable aircraft response data, as evident from the numerical estimates in Table 1 and the response match in Fig. 4, validates the various algorithms and allows us to address the practical aspects while analyzing the flight data.

X-31A Flight Data

The X-31A experimental aircraft is a highly control augmented fighter with enhanced maneuverability, the uncontrolled basic aircraft being aerodynamically unstable in distinct flight regimes. The system identification methods are currently being applied to analyze the flight data to predict

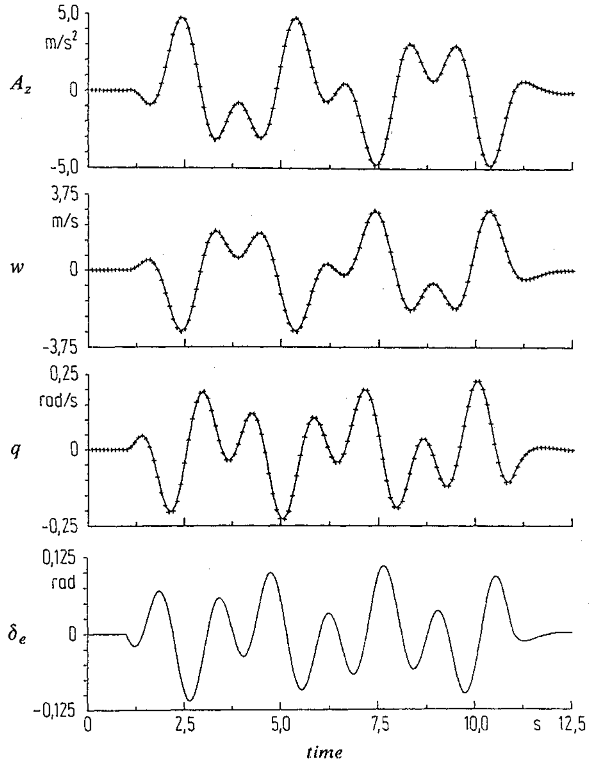

Fig. 4 Comparison of system (——) and estimated (+ + + +) responses applying a typical estimation method for unstable systems (simulated data).

aerodynamic behavior at new flight test conditions, and also to validate and update the predicted aerodynamic data base. A more detailed description of this program is found in Refs. 3 and 4. This article concentrates on the application of methods described in the foregoing section to some typical flights within the unstable flight regime.

The X-31A aircraft is known to be unstable in the longitudinal mode at low angles of attack. Five flight conditions with angle of attack varying from 6 to 19 deg, at which elevator sweep-input maneuvers were flown, are analyzed. Each sweep-input maneuver is about 30- to 40-s long. For demonstration purposes, typical results at a particular flight condition will be presented. At this selected flight condition a maneuver with two consecutive elevator doublets was also flown. This time segment with a record length of 15 s is also considered here for analysis. It should be noted that analysis of these maneuvers, considering them as a single time segments, had posed several problems and was unsuccessful up to this time. The approach adopted in the past was to partition the longer duration maneuver into several time segments, typically each less than 5-s long. This, however, necessitates estimation of unknown initial conditions for each segment separately, thereby increasing the total number of parameters being estimated. In many cases it turns out that the number of initial conditions being estimated far exceeds the actual aerodynamic derivatives in which one is really interested, making the approach laborious and time consuming.

Consistent with the aim of this article, which is an evaluation of the estimation methods on unstable aircraft flight data, no attempt is made either to justify or validate the postulated aerodynamic model. For the identification of the longitudinal aerodynamic derivatives, the following model, taken from Ref. 3, is used:

State equations

$$\dot V = -\frac{\bar q S}{m} C_D + \frac{1}{m}(X_{\text{eng}} \cos\alpha + Z_{\text{eng}} \sin\alpha) - g\sin(\theta - \alpha) \tag{27a}$$

$$\dot\alpha = -\frac{\bar q S}{mV} C_L + \frac{1}{mV}(-X_{\text{eng}} \sin\alpha + Z_{\text{eng}} \sin\alpha) + \frac{g}{V}\cos(\theta - \alpha) + q \tag{27b}$$

$$\dot q = \frac{\bar q S \bar c}{I_y} C_m^{CG} + \frac{1}{I_y} M_{\text{eng}}^{CG} \tag{27c}$$

$$\dot\theta = q \tag{27d}$$

with

$$C_m^{CG} = C_m^{AC} - C_X \frac{z_{CG}}{\bar c} + C_Z \frac{x_{CG}}{\bar c} \tag{28}$$

$$C_X = -C_D \cos\alpha + C_L \sin\alpha \tag{29a}$$

$$C_Z = -C_D \sin\alpha - C_L \cos\alpha \tag{29b}$$

The lift, drag, and pitching moment coefficients referred to the aerodynamic center are defined as

$$C_D = C_D^* + C_{D\alpha}(\alpha - \alpha^*) + C_{D\delta e}(\delta_e - \delta_e^*) \tag{30a}$$

$$C_L = C_L^* + C_{L\alpha}(\alpha - \alpha^*) + C_{L\delta e}(\delta_e - \delta_e^*) \tag{30b}$$

$$C_m^{AC} = C_m^* + C_{m\alpha}(\alpha - \alpha^*) + C_{mq}(q - q^*)(\bar c/2V) + C_{m\delta e}(\delta_e - \delta_e^*) + C_{m\delta\text{can}}(\delta_{\text{can}} - \delta_{\text{can}}^*) \tag{30c}$$

where the asterisk denotes the trim values of the corresponding variable. The model is formulated in terms of the trim values to avoid high correlation between the estimates of the various aerodynamic parameters.[3]

Observation equations

$$N_{xm} = \left(\frac{\bar q S}{m} C_X + \frac{X_{\text{eng}}}{m} - x_a q^2 + z_a \dot q\right)\Big/ g + \Delta N_x \tag{31a}$$

$$N_{zm} = \left(\frac{\bar q S}{m} C_Z + \frac{Z_{\text{eng}}}{m} - x_a \dot q + z_a q^2\right)\Big/ g + \Delta N_z \tag{31b}$$

$$V_m = \sqrt{(u + z_V q)^2 + (w - x_V q)^2} \tag{31c}$$

$$\alpha_m = \tan^{-1}\left(\frac{w - x_\alpha q}{u + z_\alpha q}\right) + \Delta\alpha \tag{31d}$$

$$q_m = q + \Delta q \tag{31e}$$

$$\theta_m = \theta \tag{31f}$$

with $u = V\cos\alpha$ and $w = V\sin\alpha$. The sensor positions relative to the c.g. are denoted by (x_a, z_a), (x_V, z_V), and (x_α, z_α), respectively. The sensor biases ΔN_x, ΔN_z, $\Delta\alpha$, and Δq are determined from a preceding data compatibility check.

The task is to estimate both the unknown initial conditions and the following aerodynamic parameters:

$$\Theta^T = (C_D^*, C_{D\alpha}, C_{D\delta e}, C_L^*, C_{L\alpha}, C_{L\delta e}, C_m^*, C_{m\alpha}, C_{mq}, C_{m\delta e}) \tag{32}$$

As pointed out in Ref. 3, the flight control law results in a canard deflection highly correlated to angle of attack. Due to this reason, and also because independent canard deflection was not possible, estimation of canard effectiveness $C_{m\delta\text{can}}$ is not possible, and therefore this parameter was kept fixed at the predicted value.

Table 2 Estimates from doublet-input maneuver of X-31A (flight test)

Parameter	Predicted value	Regression analysis	Equation decoupling	Output error with artificial stabilization	Multiple shooting	Filter error
C_D^*	0.046	0.039	0.039	0.035	0.040	0.035
$C_{D\alpha}$	0.543	0.421	0.399	0.428	0.390	0.425
$C_{D\delta e}$	0.138	0.103	0.109	0.160	0.177	0.141
C_L^*	0.403	0.365	0.370	0.339	0.376	0.340
$C_{L\alpha}$	3.057	2.661	2.709	2.688	2.637	2.572
$C_{L\delta e}$	1.354	0.863	1.074	0.988	1.110	1.006
$C_{m\alpha}$	0.119	0.217	0.227	0.238	0.252	0.218
C_{mq}	−1.650	0.137	−0.374	−0.871	−2.595	−2.124
$C_{m\delta e}$	−0.571	−0.467	−0.490	−0.504	−0.503	−0.485

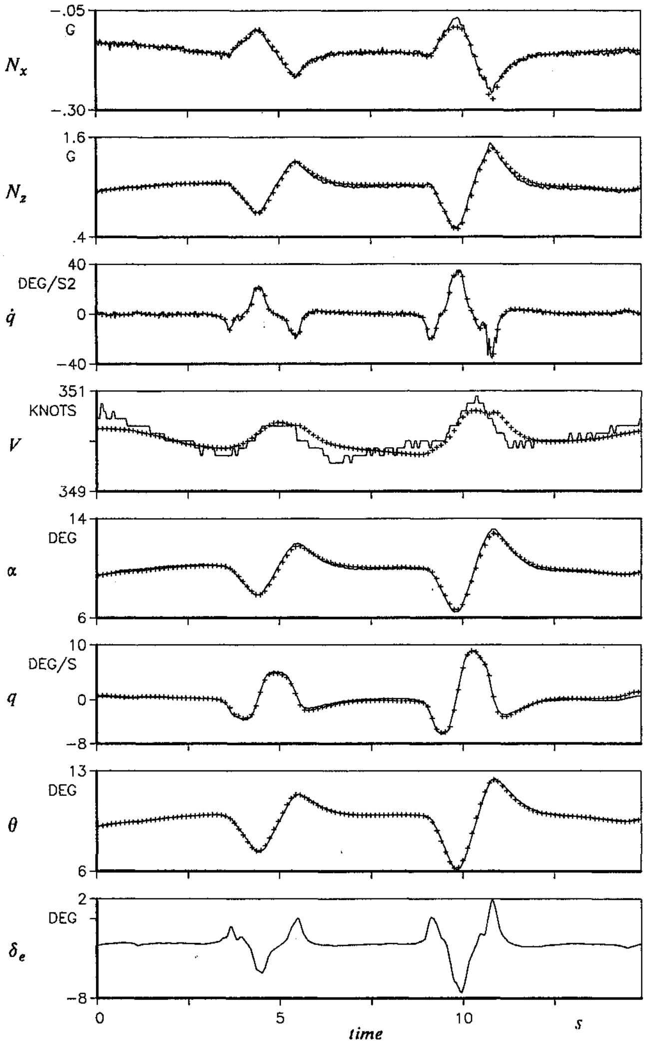

Fig. 5 Flight measured (——) and estimated (+ + + +) responses applying equation decoupling method to a doublet-input maneuver of X-31A aircraft.

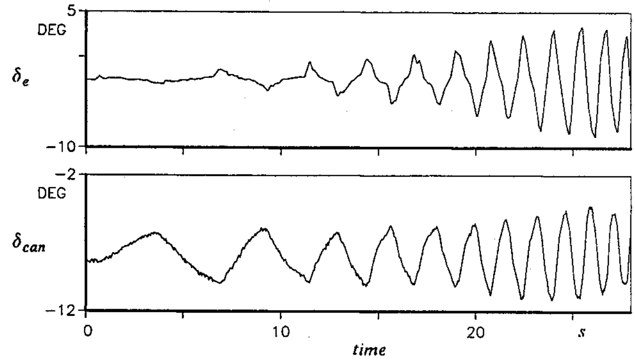

Fig. 6 Flight measured elevator and canard deflections during a sweep-input maneuver of X-31A aircraft.

Analysis of Doublet-Input Maneuver

The model postulated in Eqs. (27–31) pertains to the motion of the uncontrolled basic aircraft. The parameter estimates obtained by applying the five estimation methods to a 15-s long time segment are provided in Table 2. In this maneuver two consecutive pitch doublet inputs were applied. The predicted values for the various derivatives are provided in Table 2 for reference purposes.[3]

The regression startup method, starting from zero values of all the derivatives, converged fast without any numerical difficulties. The match, not shown in this article, for the accelerations and angle of attack was found to be good, however, that for the pitch rate and pitch angle showed slightly larger deviations. This qualitative result is also corroborated by the numerical values in Table 2, showing better estimates for the lift and drag derivatives than those for pitching moment derivatives.

The equation decoupling method converged well, yielding the time responses shown in Fig. 5. The overall agreement between the flight measured and estimated responses is seen to be good. The application of the output error method was possible only with the aid of artificial stabilization. It is usually desirable to keep the stabilization as low as possible. In the absence of any better understanding, the fit error in all the states was fed back to the corresponding states with a fixed factor of 0.05. It implies a diagonal stabilization matrix in Eq. (19). The multiple shooting method was applied with a equidistant grid, choosing every 40th data point as a grid point. The convergence was monotone and fast, requiring four iterations. The match between the flight measured and estimated responses obtained by applying the output error and multiple shooting method was qualitatively the same as that in Fig. 5.

In the case of the filter error method, two options were investigated. In the first case, keeping the process noise distribution matrix $F \approx 0$, the nonzero covariance option was invoked.[6] This provided the numerical estimates shown in Table 2. The response match was once again qualitatively the same as that in Fig. 5. This choice ensures that modeling errors are not accounted for through estimation of the process noise distribution matrix. The convergence was monotone and rapid, requiring 5–6 iterations. In the second case, the process noise matrix is simultaneously estimated. This yielded a response match which was almost perfect, and the numerical values were essentially the same as those from the nonzero covariance option.

The study of numerical values in Table 2 indicates that the majority of estimates from the various methods agree fairly well among themselves. The estimates of the pitch damping derivative C_{mq} show somewhat larger scatter. This is attributed to the poor identifiability of this derivative from unstable aircraft flight data. The automatic controller reacts instantaneously to any change in pitch rate suppressing the transient

Table 3 Estimates from sweep-input maneuver of X-31A (flight test)

Parameter	Predicted value	Regression analysis	Equation decoupling	Output error with artificial stabilization	Multiple shooting	Filter error
C_D^*	0.046	——	0.039	0.037	0.038	0.038
$C_{D\alpha}$	0.543	——	0.375	0.463	0.388	0.395
$C_{D\delta e}$	0.138	——	0.102	0.150	0.094	0.113
C_L^*	0.403	——	0.370	0.357	0.360	0.376
$C_{L\alpha}$	3.057	——	2.747	2.649	1.869	2.573
$C_{L\delta e}$	1.354	——	1.053	1.261	1.244	1.028
$C_{m\alpha}$	0.119	——	0.266	0.181	0.187	0.237
C_{mq}	−1.650	——	−1.676	−5.673	−3.083	−3.176
$C_{m\delta e}$	−0.571	——	−0.516	−0.436	−0.505	−0.503

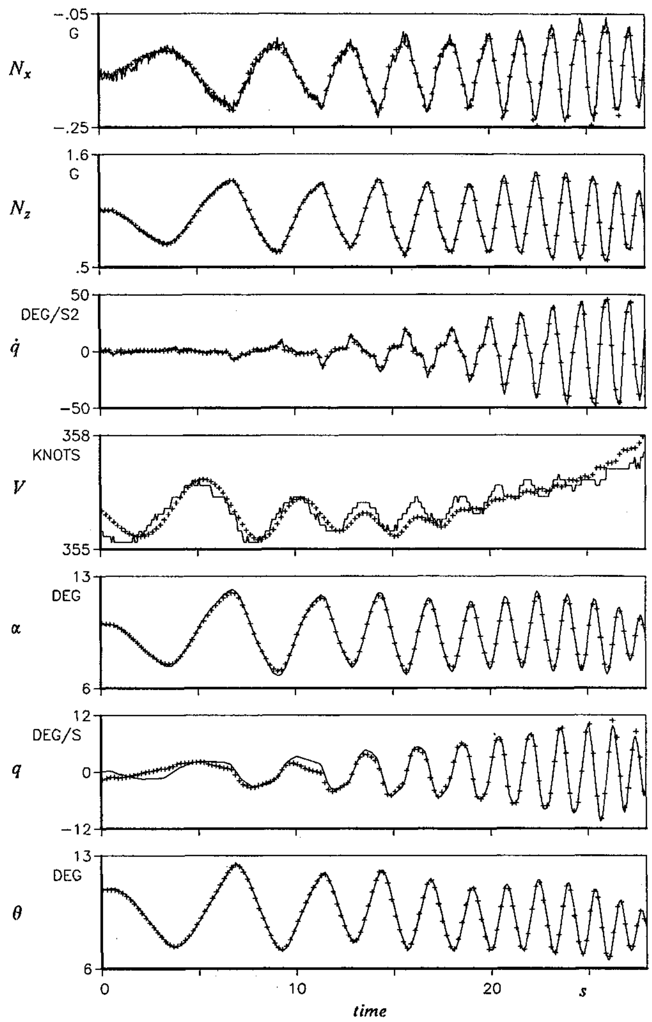

Fig. 7 **Flight measured (——) and estimated (+ + + +) responses applying equation decoupling method to a sweep-input maneuver of X-31A aircraft.**

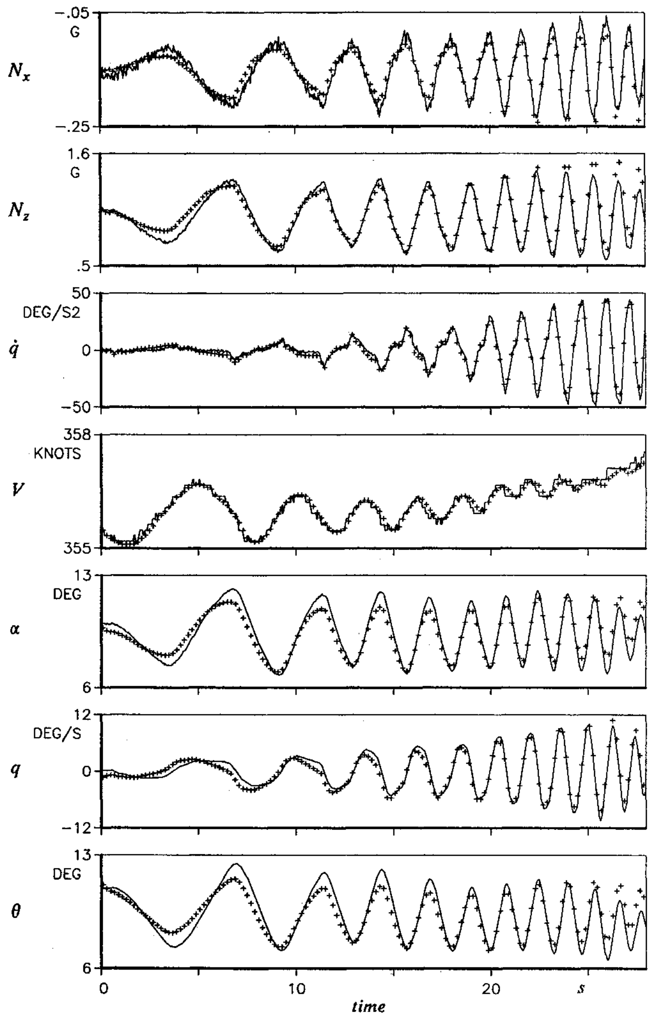

Fig. 8 **Flight measured (——) and estimated (+ + + +) responses applying output error method with artificial stabilization to a sweep-input maneuver of X-31A aircraft.**

as well as oscillatory motion due to natural damping, and thus reducing the information content in the flight data required to estimate this derivative consistently. On the other hand, the other derivatives, e.g., the effectiveness of the trailing-edge flaps, are estimated more consistently.

Analysis of Sweep-Input Maneuver

The investigations are now extended to analyze a sweep-input maneuver of 30-s duration at the same flight condition. The trailing-edge and canard deflections, which are the input variables for the postulated model in Eqs. (27–31), are shown in Fig. 6. The results of parameter estimation are provided in Table 3.

Application of the regression startup method, although it did not encounter any numerical problems, was not successful. The resulting match between the flight measured and estimated responses was unacceptable. The estimates were physically unrealistic. No specific reason could be found for the failure of this method. On the other hand, the equation decoupling method performed better, yielding the response match shown in Fig. 7. The overall agreement between the flight measured and estimated responses is considered acceptable.

Application of the output error method with artificial stabilization required some consideration. As in the previous case, the fit errors in all the states were fed back to the corresponding state variables. In order to analyze this maneuver as a single time segment, a feedback factor of at least 0.055 was necessary. This choice of the stabilization yields the estimates provided in Table 3 and the match shown in Fig. 8. Some discernible deviations are noticeable. Any further reduction in the stabilization led to numerical overflow problems. On the other hand, any further increase in the stabilization does lead to a better match, however, affecting the estimates of the aerodynamic derivatives. It is, in general, desirable to keep the artificial stabilization as low as possible. The numerical values, although appearing to be physically meaningful, do not fall within the trend of flight estimates obtained in this investigation applying other estimation methods as well as those already reported in Ref. 3 from other flight maneuvers.

In the case of the multiple shooting method several different grids were tried out, attempting to match either the low-frequency or high-frequency dynamics, or even a combination of the two. For example, 26 nonequidistant grid points were chosen to match high dynamic response. This choice yielded estimates shown in Table 3. Although the numerical values appear plausible, some detectable differences compared to the filter error and equation decoupling methods are noticed. Moreover, the response match also showed significant deviations. A different choice of the grid also led to similar discrepancies. In none of the attempts were entirely satisfactory results obtained by applying this method, although sufficient engineering effort was applied.

As in the previous case, the convergence of the filter error method was fast and encountered no numerical difficulties. The match between the flight measured and estimated responses was good, and therefore not explicitly shown. The resulting estimates, as seen from Table 3, compare well with those obtained from the equation decoupling method. Once again, larger scatter in the estimates of C_{m_q} is observed.

It is interesting to note that no significant correlations were noticed among the parameter estimates. The standard derivations (Cramer-Rao bounds) were of the order of 2–5%. Even after accounting for a fudge factor of 5, the theoretical estimates of the accuracies are considered acceptable, and not explicitly included in Tables 2 and 3.

The estimates obtained by applying the filter error method and equation decoupling method to the doublet-input maneuver, Table 2, and to the sweep-input maneuver, Table 3, show the same trends and scatter. These flight estimates, although showing some detectable differences compared to the predicted values, fall within the trends already observed in the other investigations. As can be deduced, the output error method with artificial stabilization and the multiple shooting method required more engineering effort, and still the performance was not entirely satisfactory.

Programming Considerations

In order to compare the complexities of modifying the existing estimation software or writing new programs, it is reasonable to assume that software for the output error algorithm is available. Incorporation of the regression startup is a trivial exercise. Many estimation programs already incorporate this option. Application of the equation decoupling method does not warrant any modifications. Only the postulated model needs to be properly coded. Including the artificial stabilization in the output error method requires some software modification. These changes are comparatively minor and can be incorporated without much developmental time. On the other hand, the filter error algorithm may not be universally available. New development of such software and its validation is time consuming and costly. However, a few estimation software packages do exist,[10,16] and experience gained can be used to minimize the developmental costs. The multiple shooting method for aircraft parameter estimation is a relatively new approach. The estimation software tested out here was only a basic version. The programming complexity of the multiple shooting method with condensation algorithm is of the same order as that of a filter error method. Although the examples reported help to establish the feasibility of the approach, further refinements are necessary to make it more easily applicable in a routine manner.

Concluding Remarks

In this article an overview has been made of the parameter estimation methods applicable to unstable aircraft. The emphasis has been placed on analyzing longer duration maneuvers without time segmenting. The various methods were validated on the simulated response data of an unstable aircraft, and were found to perform well. Their application to X-31A aircraft flight data in unstable flight regime, however, brought out the differences. In one case, namely the 15-s-long doublet-input maneuver, all of the methods provided comparable results. The convergence was monotone and no numerical difficulties were encountered. In the other case, namely the 30-s-long sweep-input maneuver, only the filter error and equation decoupling methods performed satisfactorily. The more routinely adopted output error method with artificial stabilization and also the multiple shooting method required more engineering effort, and still provided estimation results which could not be completely resolved. Thus, it can be concluded that the other two methods offer a less laborious and attractive alternative to analyze longer duration maneuvers such as the sweep-input maneuvers reported, or acceleration, deceleration, and level turn maneuvers.

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